OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4724

Core Mathematics 4

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

[Turn over

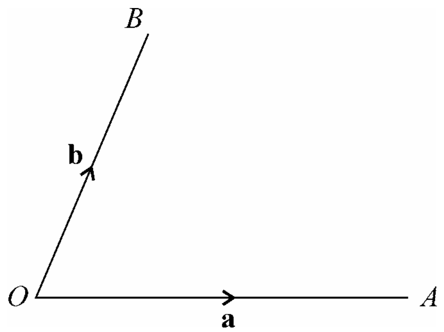
1 Find the quotient and remainder when $x^4 + 1$ is divided by $x^2 + 1$. [4]

2 **(i)** Expand $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3. [4]

(ii) State the set of values for which the expansion in part **(i)** is valid. [1]

3 Find $\int_0^1 x\mathrm{e}^{-2x}\,\mathrm{d}x$, giving your answer in terms of e. [5]

4



As shown in the diagram the points A and B have position vectors **a** and **b** with respect to the origin O.

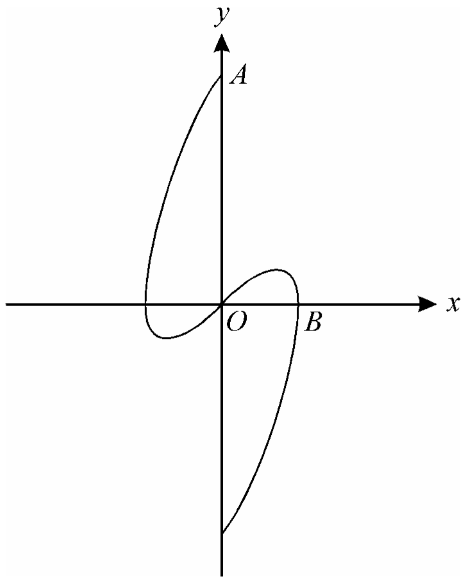
(i) Make a sketch of the diagram, and mark the points C, D and E such that $\overrightarrow{OC} = 2\mathbf{a}$, $\overrightarrow{OD} = 2\mathbf{a} + \mathbf{b}$ and $\overrightarrow{OE} = \frac{1}{3}\overrightarrow{OD}$. [3]

(ii) By expressing suitable vectors in terms of **a** and **b**, prove that E lies on the line joining A and B. [4]

5 **(i)** For the curve $2x^2 + xy + y^2 = 14$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(ii) Deduce that there are two points on the curve $2x^2 + xy + y^2 = 14$ at which the tangents are parallel to the x-axis, and find their coordinates. [4]

6



The diagram shows the curve with parametric equations

$$x = a\sin\theta, \quad y = a\theta\cos\theta,$$

where a is a positive constant and $-\pi \leqslant \theta \leqslant \pi$. The curve meets the positive y-axis at A and the positive x-axis at B.

(i) Write down the value of θ corresponding to the origin, and state the coordinates of A and B. [3]

(ii) Show that $\dfrac{dy}{dx} = 1 - \theta\tan\theta$, and hence find the equation of the tangent to the curve at the origin. [6]

7 The line L_1 passes through the point $(3, 6, 1)$ and is parallel to the vector $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$. The line L_2 passes through the point $(3, -1, 4)$ and is parallel to the vector $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

(i) Write down vector equations for the lines L_1 and L_2. [2]

(ii) Prove that L_1 and L_2 intersect, and find the coordinates of their point of intersection. [5]

(iii) Calculate the acute angle between the lines. [4]

8 Let $I = \displaystyle\int \frac{1}{x(1+\sqrt{x})^2}\,dx$.

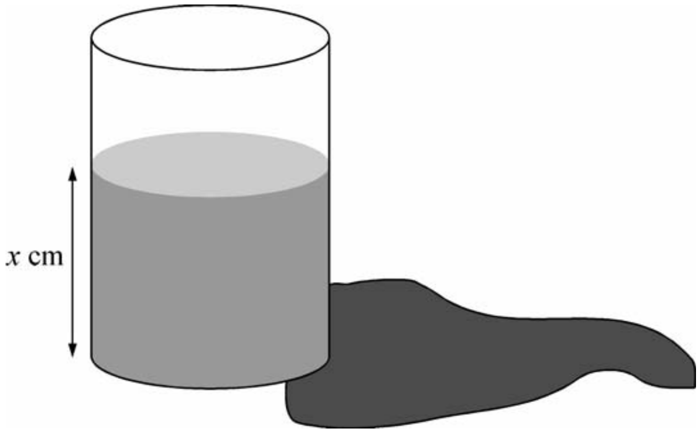
(i) Show that the substitution $u = \sqrt{x}$ transforms I to $\displaystyle\int \frac{2}{u(1+u)^2}\,du$. [3]

(ii) Express $\dfrac{2}{u(1+u)^2}$ in the form $\dfrac{A}{u} + \dfrac{B}{1+u} + \dfrac{C}{(1+u)^2}$. [5]

(ii) Hence find I. [4]

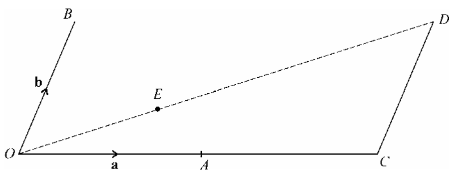
[Turn over

9



A cylindrical container has a height of 200 cm. The container was initially full of a chemical but there is a leak from a hole in the base. When the leak is noticed, the container is half-full and the level of the chemical is dropping at a rate of 1 cm per minute. It is required to find for how many minutes the container has been leaking. To model the situation it is assumed that, when the depth of the chemical remaining is x cm, the rate at which the level is dropping is proportional to $\sqrt{x}$.

Set up and solve an appropriate differential equation, and hence show that the container has been leaking for about 80 minutes. [11]

1	$\dfrac{x^4+1}{x^2+1} = x^2 - 1 + \dfrac{2}{x^2+1}$	B1 M1 A1 A1 **4** $\boxed{4}$	For correct leading term x^2 in quotient For evidence of correct division process For correct quotient x^2-1 For correct remainder 2
2	**(i)** $(1-2x)^{-\frac{1}{2}} = 1 + (-\frac{1}{2})(-2x) + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-2x)^2 + $ $+\dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-2x)^3 + \ldots$ $= 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3$	M1 A1 A1 A1 **4**	For 2nd, 3rd or 4th term OK (unsimplified) For $1+x$ correct For $+\frac{3}{2}x^2$ correct For $+\frac{5}{2}x^3$ correct
	(ii) Valid for $\lvert x\rvert < \frac{1}{2}$	B1 **1** $\boxed{5}$	For any correct expression(s)
3	$\int_0^1 x\mathrm{e}^{-2x}\,\mathrm{d}x = \left[-\frac{1}{2}x\mathrm{e}^{-2x}\right]_0^1 - \int_0^1 -\frac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$ $= \left[-\frac{1}{2}x\mathrm{e}^{-2x} - \frac{1}{4}\mathrm{e}^{-2x}\right]_0^1$ $= \frac{1}{4} - \frac{3}{4}\mathrm{e}^{-2}$	M1 A1 M1 M1 A1 **5** $\boxed{5}$	For attempt at 'parts' going the correct way For correct terms $-\frac{1}{2}x\mathrm{e}^{-2x} - \int -\frac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$ For consistent attempt at second integration For correct use of limits throughout For correct (exact) answer in any form
4	**(i)** 	B1 B1 B1√ **3**	For C correctly located on sketch For D correctly located on sketch For E correctly located wrt O and D
	(ii) $\overrightarrow{AE} = \frac{1}{3}(2\mathbf{a}+\mathbf{b}) - \mathbf{a} = \frac{1}{3}(\mathbf{b}-\mathbf{a})$ Hence AE is parallel to AB i.e. E lies on the line joining A to B	M1 A1 A1 A1 **4** $\boxed{7}$	For relevant subtraction involving $\overrightarrow{OE}$ For correct expression for $(\pm)\overrightarrow{AE}$ or $\overrightarrow{EB}$ For correct recognition of parallel property For complete proof of required result
5	**(i)** $4x + x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y + 2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ Hence $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{4x+y}{x+2y}$	B1 B1 M1 A1 **4**	For correct terms $x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y$ For correct term $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$ For solving for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ For any correct form of expression
	(ii) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow y = -4x$ Hence $2x^2 + (-4x^2) + (-4x)^2 = 14$ i.e. $x^2 = 1$ So the two points are $(1, -4)$ and $(-1, 4)$	M1 M1 A1 A1 **4** $\boxed{8}$	For stating or using their $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ For solving simultaneously with curve equn For correct value of x^2 (or y^2) For both correct points identified

6 **(i)** $\theta = 0$ at the origin	B1	For the correct value
A is $(0, a\pi)$	B1	For the correct y-coordinate at A
B is $(a, 0)$	B1 **3**	For the correct x-coordinate at B
(ii) $\dfrac{dx}{d\theta} = a\cos\theta$	B1	For correct differentiation of x
$\dfrac{dy}{d\theta} = a(\cos\theta - \theta\sin\theta)$	M1	For differentiating y using product rule
Hence $\dfrac{dy}{dx} = \dfrac{\cos\theta - \theta\sin\theta}{\cos\theta} = 1 - \theta\tan\theta$	M1	For use of $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \Big/ \dfrac{dx}{d\theta}$
	A1	For given result correctly obtained
Gradient of tangent at the origin is 1	M1	For using $\theta = 0$
Hence equation is $y = x$	A1 **6**	For correct equation
	[9]	
6 **(i)** $L_1 : \mathbf{r} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$	M1	For correct RHS structure for either line
$L_2 : \mathbf{r} = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	A1 **2**	For both lines correct
(ii) $3 + 2s = 3 + t,\ 6 + 3s = -1 - 2t,\ 1 - s = 4 + t$	M1	For at least 2 equations with two parameters
First pair of equations give $s = -1, t = -2$	M1	For solving any relevant pair of equations
	A1	For both parameters correct
Third equation checks: $1 + 1 = 4 - 2$	A1	For explicit check in unused equation
Point of intersection is $(1, 3, 2)$	A1 **5**	For correct coordinates
(iii) $2\times1 + 3\times(-2) + (-1)\times1 = (\sqrt{14})(\sqrt{6})\cos\theta$	B1	For scalar product of correct direction vectors
	B1	For correct magnitudes $\sqrt{14}$ and $\sqrt{6}$
	M1	For correct process for $\cos\theta$ with *any* pair of vectors relevant to these lines
Hence acute angle is $56.9°$	A1 **4**	For correct acute angle
	[11]	
8 **(i)** $I = \displaystyle\int \frac{1}{u^2(1+u)^2} \times 2u\,du = \int \frac{2}{u(1+u)^2}\,du$	M1	For any attempt to find $\dfrac{dx}{du}$ or $\dfrac{du}{dx}$
	A1	For '$dx = 2u\,du$' or equivalent correctly used
	A1 **3**	For showing the given result correctly
(ii) $2 \equiv A(1+u)^2 + Bu(1+u) + Cu$	M1	For correct identity stated
$A = 2$	B1	For correct value stated
$C = -2$	B1	For correct value stated
$0 = A + B$ (e.g.)	A1	For any correct equation involving B
$B = -2$	A1 **5**	For correct value
(iii) $2\ln u - 2\ln(1+u) + \dfrac{2}{1+u}$	B1√	For $A\ln u + B\ln(1+u)$ with their values
	B1√	For $-C(1+u)^{-1}$ with their value
Hence $I = \ln x - 2\ln(1+\sqrt{x}) + \dfrac{2}{1+\sqrt{x}} + c$	M1	For substituting back
	A1 **4**	For completely correct answer (excluding c)
	[12]	

[Turn over

9	$\frac{dx}{dt} = -k\sqrt{x}$	M1		For use of derivative for rate of change
		A1		For correct equation (neg sign optional here)
	$x = 100$ and $\frac{dx}{dt} = -1 \Rightarrow k = 0.1$	M1		For use of data and their DE to find k
	Hence equation is $\frac{dx}{dt} = -0.1\sqrt{x}$	A1		For any form of correct DE
	$\int x^{-\frac{1}{2}}\,dx = -0.1\int dt \Rightarrow 2x^{\frac{1}{2}} = -0.1t + c$	M1		For separation and integration of both sides
		A1		For $2x^{\frac{1}{2}}$ correct
		A1√		For $(\pm)kt$ correct (the numerical evaluation of k may be delayed until after the DE is solved)
		B1		For one arbitrary constant included (or equivalent statement of both pairs of limits)
	$x = 200, t = 0 \Rightarrow c = 2\sqrt{200}$	M1		For evaluation of c
	So when $x = 100$, $2\sqrt{100} = -0.1t + 2\sqrt{200}$	M1		For evaluation of t
	i.e. $t = 82.8$	A1	**11**	For correct value 82.8 (minutes)
			$\boxed{\mathbf{11}}$	